Spectrum policy and competition in mobile data

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Abstract

Radio spectrum is an essential input for delivering mobile communication services. Traditionally blocks of licensed spectrum have been exclusively controlled by its buyer. However, in recent years there has been an increasing focus by regulators on developing policies and practices which ensure a more efficient utilization of the spectrum resource, among which spectrum sharing is one of the most prominent. The current paper analyzes the effect of sharing radio spectrum. Using a simple duopoly model we study how spectrum sharing is likely to affect the competition between two mobile operators who choose the price and data volume of their offers. We show that spectrum sharing can have adverse effects: First, it creates an incentive for producers to strategically increase the load in their networks in order to weaken their competitors, since unused capacity increases the competitor’s capacity. Increasing the network load also increases network congestion, and hence leads to suboptimal equilibrium product quality. Second, consumer surplus decreases and industry profit increases for a wide range of parameter values in the model. In other words, spectrum sharing could lead to a transfer from consumers to producers: the growth in quality from spectrum sharing does not match the increase in equilibrium prices.

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1 Introduction

After the introduction of the smartphone in 2008 there has been a tremendous growth in the demand for mobile data. This growth is expected to continue for many years to come.\(^1\) At the same time the supply side, the amount of spectrum licensed for transmitting mobile data, will not experience a comparable growth. As a response to this development new technologies and regulatory remedies are being considered which ensure a more efficient utilization of the spectrum resource. For instance The European Unioun (EU 2012) has mandated its member states to foster spectrum sharing. This is now gradually being implemented by member states.\(^2\) Moreover, there are examples of countries where clauses on spectrum sharing are already included in the 4G license text.\(^3\)

An important argument for such a regime is that it will result in a welfare improvement since a scarce resource, spectrum, is utilized more efficiently. There is however one important aspect missing in this line of argument: the strategic effect of introducing spectrum sharing, i.e. how the firms’ strategic incentives regarding spectrum utilization, and hence the market equilibrium, is affected by spectrum sharing. To our knowledge, this paper is the first to study this question.

Using a simple duopoly model where competing networks choose the prices and qualities of their services we find a rather counter-intuitive result: spectrum sharing could lead to higher end-user prices and lower consumer surplus. The intuition for the adverse effects of spectrum sharing is directly related to the strategic effects: it causes the firms’ capacities to become interrelated since unused capacity can be utilized by a rival. This generates a strategic incentive to increase network load in order to weaken one’s rival; however increased network load also creates congestion, which affects product quality negatively. The paper also derives the welfare maximizing marginal price on unused spectrum and show that within our

\(^1\) According to Cisco (2013) global mobile data traffic grew 70 percent in 2012 and for Western Europe by 44%. Cisco projects an annual growth rate of 50% for the period 2013 – 2018.

\(^2\) A timeline for UK implementation is found in figure 14 in Ofcom (2013).

\(^3\) The recent spectrum auction in Norway contains a caveat on spectrum sharing by the use of cognitive radio. See clause 2 in NPT 2013.
model this price coincides with the profit maximizing price of the spectrum owners. The landmark contribution when it comes to spectrum management is Coase (1959), which argues that the market mechanism will ensure an optimal use of spectrum, provided that ownership rights are allocated and transaction costs are sufficiently low. Since 1959, regulating authorities in the US, Europe, and most other countries of the world have gradually implemented this prescription by the means of spectrum auctions.

There are several other papers analyzing different aspects of optimal spectrum management, for instance different hybrid regimes of licensed and unlicensed spectrum (e.g. Freyens, 2009), mechanisms for avoiding the "tragedy of the commons" in unlicensed spectrum (e.g. Bykowsky et al., 2010), trading in secondary spectrum markets (e.g. Bykowsky, 2003; Crocioni, 2009). There are also papers which study imperfect competition for end-users in secondary spectrum markets (e.g. Kim et al., 2011 and Duan and Shou, 2010). The unique contribution of our paper is develop a model of imperfect competition that allows for an analysis of how the strategic incentives of firms and welfare are influenced by the introduction of spectrum sharing through the use of cognitive radio.

From here the paper proceeds as follows. Section 2 and 3 presents some background on how network congestion and consumer demand will be modeled. Section 4 develops the welfare maximizing benchmark while section 5 solves the model of competing networks under the commons regime. In section 6 we analyze the game with spectrum trading and in section 7 solutions are compared based on a numerical example. Finally, in section 8 we conclude.

2 Modeling network congestion

For a given capacity in a mobile network, there is a trade-off between the number of customers served and average waiting time. We formalize this by deploying the widely used $M/M/1$ queing model.\footnote{See for instance Choi and Kim (2010).}

Consider a network $i$ with capacity $K_i$, number of customers $s_i$, and data volume
per customer equal to \( y_i \). Using the \( M/M/1 \) queuing model, the average waiting time on network \( i \) is

\[
\bar{w}_i = \frac{1}{K_i - s_i y_i}.
\] (1)

Following Bourreau et al. (2013), we assume that quality is the inverse of waiting time, i.e.

\[
\bar{q}_i = \frac{1}{w_i} = K_i - s_i y_i.
\] (2)

Now assume that there are two networks, \( i = 1, 2 \), whose capacities are pooled. The traffic from both networks enter the same queue, on a first come first serve basis. Again using the \( M/M/1 \) queuing model we get average waiting time

\[
\bar{w} = \frac{1}{K_i + K_j - s_i y_i - s_j y_j}.
\] (3)

Notice that if the networks are identical, then \( w = 1/2\bar{w}_1 = 1/2\bar{w}_2 \), hence average waiting time is halved by pooling. The quality of the shared network is

\[
\bar{q} = K_i + K_j - s_i y_i - s_j y_j.
\] (4)

Now consider the following rewriting of network \( i \)'s quality:

\[
q_i = K_i - s_i y_i + \theta (K_j - s_j y_j),
\] (5)

we now have that

\[
q_i = \begin{cases} 
\bar{q}_i & \text{if } \theta = 0 \\
\bar{q} & \text{if } \theta = 1 . 
\end{cases}
\] (6)

Using this formalization, we interpret \( \theta \in [0, 1] \) as the degree of spectrum sharing.$^5$

### 3 Consumer utility and demand

We consider a market that has two competing networks, \( i = 1, 2 \), selling network access to consumers. The consumers’ utility of using the mobile network depends on the quantity and quality offered, i.e. data volume (\( y_i \)) and average download

\[ ^5 \text{Alternatively } \theta \text{ could be interpreted as the technological efficiency of the spectrum sharing technology.} \]
speed \((q_i)\), respectively. It is assumed that utility increase in both of these product features. We can accordingly formulate a utility function: \(u_i(y_i, q_i)\). In the analysis below we assume a particular parametric form for this utility function:

\[
  u_i = v + y_i q_i = v + y_i \left( K_i - s_i y_i + \theta (K_j - s_j y_j) \right),
\]

(7)

where \(v\) denotes a base utility of the service. This functional form captures a crucial feature of this product: quantity and quality are “complements” – quantity is worthless with zero quality, and vice versa. Note that this functional form is a special case of a Cobb Douglas function.

The sellers are located at opposite ends of a Hotelling line of unit length on which the consumers are uniformly distributed.\(^6\) All the consumers buy access from one of the two sellers (full market coverage). Even though network access could be considered a homogenous good, we assume that the consumers are horizontally differentiated based on features with the sellers that are exogenous to the model, such as brand, handset offers or premium services. Let \(x\) denote the distance from a consumer to seller. The consumer’s net utility when buying from network is:

\[
  U_i = u_i - p_i - tx,
\]

(8)

The indifferent consumer is placed at the point where the net utilities are equal for both networks. Without loss of generality we let seller 1 and 2 be situated at \(x = 0\) and \(x = 1\), respectively. The indifferent consumer is placed at

\[
  x = \frac{1}{2} + \frac{u_1 - u_2 - (p_1 - p_2)}{2t}. \tag{9}
\]

Note that since the market size is normalized to one, the market shares \(s_1\) and \(s_2\) must be equal to \(x\) and \(x - 1\). We therefore substitute for \(s_1\) and \(s_2\), and then solve for \(x\) in order to express the placement of the indifferent consumer as a function of variables and parameters. Futhermore, in order to obtain closed form solutions and keep the model tractable, we assume that the networks are symmetrical in terms of capacity, \(K_1 = K_2 = K\). We then find

\[
  x = \frac{t + K (1 + \theta) (y_1 - y_2) - (p_1 - p_2) - \theta y_1 y_2 + y_2^2}{2t + y_1^2 + y_2^2 - 2\theta y_1 y_2}, \tag{10}
\]

where we have used that: \(u_i = v + y_i (K - s_i y_i + \theta (K - s_j y_j))\).

\(^6\)Known as the Hotelling model after Hotelling (1929).
4 First best benchmark

We are considering a symmetric Hotelling model and consumers are identical in the volume dimension, hence a welfare maximizing solution must be characterized by consumers of mass $1/2$ on the two networks. Furthermore, we normalize fixed and marginal costs to zero. Finally, the welfare maximizing data volume (bucket size) $y$ is defined by $y = \max_y y (K - s_i y + \theta (K - (1 - s_i) y))$ since prices only serve as transfers in the current model. Since $s_i = s_j = 1/2$ in first best, it can easily be verified that the bucket size that maximizes total welfare is

$$y = K.$$ \hfill (11)

By setting this bucket size one obtains the optimal balance between bucket size $y$ and quality $q_i = K - s_i y_i + \theta (K - s_j y_j)$. It is notable that the optimal bucket size is independent of $\theta$. Hence, within our model, optimal bucket size is independent of whether spectrum is shared, and also independent of the effectiveness of spectrum sharing. By inserting the optimal bucket size $y = K$ as well as optimal market shares $s_i = s_j = 1/2$ we find the social welfare function can be reduced to: $0.5K^2 (1 + \theta)$. Thus welfare is increasing in the degree of spectrum sharing, verifying that spectrum sharing indeed has the potential to increase welfare.

5 Spectrum commons

In this section we consider the competitive outcome under the assumption that the firms can access its rival’s unused spectrum without paying any compensation. Since fixed and variable costs are normalized to zero, the firms’ profits are:

$$\pi_1 = p_1 x,$$ \hfill (12)

$$\pi_2 = p_2 (1 - x),$$ \hfill (13)

with first order conditions for firm $i = 1, 2$ with respect to price ($p_i$) and bucket
size \( y_i \).\(^7\)

\[
\frac{\partial \pi_i}{\partial p_i} = 0, \quad (14)
\]

\[
\frac{\partial \pi_i}{\partial y_i} = 0. \quad (15)
\]

Solving for the equilibrium and applying symmetry we obtain

\[
p^* = t + (1 - \theta) y^2, \quad (16)
\]

\[
y^* = (1 + \theta) K. \quad (17)
\]

From equation (16) we see that there are two opposing effects on the equilibrium price level from increasing the degree of network sharing \((d/p^*/d\theta)\): i) a negative direct effect and ii) a positive indirect, or strategic, effect through \(y^*\). The intuition for these effects are as follows. Consider first the direct price effect \((\partial p^*/\partial \theta)\), i.e. holding \(y\) fixed: Spectrum sharing creates an incentive to compete more aggressively for consumers. The reason is that it limits the negative effects from business stealing on network load since stealing customers from the rival will free up spectrum on the competing network that can be utilized via spectrum sharing. Consider next the indirect or strategic effect on price from network sharing: Firms increase bucket size \((\partial y^*/\partial \theta > 0)\) in order to weaken their rival, which in turn softens competition. It can be shown that the equilibrium price is maximized when \(\theta = 1/3\), hence the positive indirect effect dominates for \(\theta < 1/3\) and the negative direct effect dominates for \(\theta > 1/3\). Note also that compared to a situation without spectrum sharing, equilibrium prices increase for all \(\theta < 0.618\).

From equation (17) it can easily be verified that bucket sizes increase in both capacity and spectrum sharing. Compared to the welfare maximizing bucket size found in equation (11), we see the market outcome coincides with the welfare maximum when \(\theta = 0\). Thus the market solution imply that networks become “too” crowded – there is too much congestion whenever \(\theta > 0\). Cognitive radio hence leads to a welfare distortion which is proportional to the degree to which the spectrum

\(^7\)From equation (10) we see that market shares are a function of both price and bucket size, \(x(p_1, p_2, y_1, y_2)\). For convenience we will denote market shares by \(x\).
sharing technology is efficient ($\theta$). The intuition for this result is that a seller strategically exerts a negative externality on the competitor’s capacity by increasing the size of its own data buckets.

5.1 Welfare - industry profit and consumer surplus

The firms’ profit is $\pi^* = p^* x^*$. Note that since the firms are symmetric we have that $x^* = 1/2$. This implies that the effect of spectrum sharing on industry profit is linearly proportional to the effect on the equilibrium price level since $\frac{\partial x^*}{\partial \theta} = 0$.

The effects of spectrum sharing on consumer surplus does require some more analysis. It can be found by substituting for the equilibrium bucket size ($y^*$) and price ($p^*$) into the utility function, given by equation (8). We then obtain

$$U^* = \nu + \frac{K^2 (\theta - 1)(1 + \theta)^2}{2} - t.$$ 

It can be shown that $U^*$ is minimized for $\theta = 1/3$, hence $dU^*/d\theta < (>)0$ for $\theta < (>) 1/3$. Furthermore, the net effect on consumer surplus from spectrum sharing is negative for all $\theta < 0.618$.

Finally, we consider welfare - the sum of profits and consumer surplus. Note that since we are using a Hotelling framework where market size is fixed, equilibrium price level has no welfare effect except a transfer between consumers and firms. The main determinant of welfare is hence the equilibrium quality level. It turns out that the effect on industry profit dominates the effect on consumer surplus. We hence find that welfare first increases, then decreases in $\theta$.

6 Spectrum trading

In the previous section we demonstrated that under spectrum sharing the firms would strategically increase the traffic on their networks in order to weaken their rival. One should expect that there exists a price on unused spectrum which is such that the negative externality is neutralized and the market hence implements the first best solution. In this section we will explore this possibility.
The spectrum controlled by firm \( j \) that firm \( i \) can access is given by: \( K - s_jy_j \). We will investigate the market outcome when there is a symmetric unit price \( \mu \) on this spectrum. We will assume that \( \mu \) is exogenously determined prior to the game analyzed in the previous section. The profit the firm \( i = 1, 2 \) is now:

\[
\pi_i = p_is_i + \mu(K - s_iy_i) - \mu(K - s_jy_j) .
\]

(18)

The first term on the right hand side is retail profits, the second term is revenues from selling unused spectrum and the third term is cost from buying spectrum from the rival. As we can see, revenues from selling spectrum at firm \( i \) equals the cost from buying spectrum at firm \( j \), and vice versa. Similarly to the previous section, the firms maximized their respective profits with respect to price (\( p_i \)) and bucket size (\( y_i \)). Applying symmetry we derive the following equilibrium outcomes:

\[
p^\ast = t + y^2(1 - \theta) + 2y\mu ,
\]

(19)

\[
y^\ast = K + \theta K - \mu.
\]

(20)

Comparing to the equilibrium values in the commons scenario the only difference is the last term in each of the above expressions. A benevolent regulator would set the price on shared spectrum such that the first best is implemented. Thus this price should be set such that bucket size \( y \) and hence quality is at the optimal level.

In section 3 we found that the optimal bucket size is \( y = K \), irrespective of the level of \( \theta \). From equation (20) we easily see that the welfare maximizing price on shared spectrum (\( \mu \)) is \( \mu = \theta K \).

If the firms were to determine a symmetric price on shared spectrum, they would agree on the price (\( \mu \)) that maximizes \( p^\ast \). It turns out that \( p^\ast \) is maximized when \( \mu = \theta K \), hence the first best solution and the profit maximizing solution coincides.

From (19) and (20) we see that the equilibrium price increases in \( \mu \) for given bucket size (\( y \)), and that equilibrium bucket size decreases in \( \mu \). There are thus two opposing effects on firm profits from increasing the price on shared spectrum: a positive direct effect and negative indirect effect through \( y^\ast \). Inserting for \( y^\ast \) from (20) into (19), we find that \( dp^\ast/d\mu > 0 \) for all \( 0 > \theta > 1 \). This implies that the equilibrium price levels, and firms profits will increase in \( \mu \).
When it comes to consumer surplus there are two opposing effects. On the one hand spectrum trading makes consumers worse off by causing an increase in end user prices. On the other hand spectrum trading makes consumers better off by causing an increase in the equilibrium product quality. The negative price effect dominates, thus consumer become worse off when the first best solution ($\mu = \theta K$) is implemented.

7 Comparing solutions

In the previous sections we compared a spectrum commons regime to a regime with spectrum trading. In order to get a better overview of the results, it is useful to consider some figures based on numerical examples. In this section we therefore look closer at the results derived in the previous sections by the means of some simple figures. We assume $v = 2.5$, $K = 1$ and $t = 1$. When considering spectrum trading we shall also assume that the first best price on unused spectrum is implemented, hence $\mu = \theta K$. Consider first data volume, or bucket size:
We see from the figure that the bucket size does not increase if spectrum trading is introduced, whereas bucket size will increase if spectrum commons is introduced, for any value on the effectiveness of spectrum sharing.

Consider next equilibrium price:
As in the previous illustration, the point on the vertical axis where the two curves start is equivalent to the outcome without spectrum sharing. As we can see, prices will monotonously increase if spectrum trading is introduced. Prices will also increase under a spectrum commons regime, given that $\theta < 0.618$.

In our model there are no variable costs, and the equilibrium market shares are equal to 0.5, hence equilibrium profits are $\frac{b^r}{2}$. It follows that the profit curve has the same shape as the price curve. The competing firms will accordingly always prefer a trading regime over a commons regime, and for plausible parameter values they will prefer spectrum commons over no spectrum sharing. Price and bucket size, considered separately above, have a combined effect on consumer surplus. The
effect on consumer surplus is plotted below:

Figure 4: consumer surplus as a function of the effectiveness of spectrum sharing

Compared to the situation without spectrum sharing, consumers will never benefit from spectrum sharing, provided that it is not “too” effective to share spectrum ($\theta < 0.618$). Furthermore, consumers never benefit from spectrum trading. Note that consumers and firms have opposite preference ordering over the alternative regimes. Consumers always prefer a commons regime over a trading regime, and they will for $\theta < 0.618$ typically prefer exclusive spectrum ($\theta = 0$) over spectrum commons.
Finally, we consider welfare:

As we can see, welfare will always increase if spectrum trading is introduced. Welfare also increases with the introduction of spectrum commons if $\theta < 0.618$. Furthermore, from a welfare perspective, spectrum trading is always preferable to spectrum commons. The preferences of a benevolent regulator and the competing firms accordingly coincide, whereas consumers have an opposite preference ordering over the alternative regimes. Consumers are likely to be worse off if spectrum sharing is introduced, while the competing firms benefit.

**8 Discussion and conclusion**

Regulators have already introduced caveats in spectrum rights that open up for the use of cognitive radio and other forms of spectrum sharing. In this paper we develop a model showing that spectrum sharing can have adverse effects. It also
shows that consumer surplus typically decrease due to an increase in price level that is not “matched” by an equal valuable increase in quality. We furthermore show that welfare increase for a wide range of parameter values, but that this increase is due to a transfer from consumers to producers.

By comparing the alternative regimes for spectrum sharing we have found that consumers on the one side and firms and a benevolent regulator on the other side have opposite preference orderings over alternative regimes. For plausible parameter values - low to medium efficiency in spectrum sharing - consumers prefer exclusive spectrum rights over any type of shared spectrum. If spectrum sharing is introduced consumers prefer spectrum commons over spectrum trading.

This paper is an early attempt at investigating some aspect of spectrum sharing. There are many unanswered questions, and we acknowledge that the model in this paper leaves out several important aspects. There are at least three topics that we would like to mention: i) what is the effect if capacity is made endogenous? ii) how will asymmetry in network sizes affect results?

References


